1879MNRAS..39..425A

Two Short and Easy Methods for correcting Lunar Distances.

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(*Communicated by Lord Lindsay.*)

The following methods for correcting lunar distances are equal in accuracy to those of Mendoza y Rios and Witchell, but considerably shorter and easier. As well as these methods, they are based on the well-known expression for the true distance developed by help of Taylor's Theorem, viz.—

$$(1)\quad d = d' - \Delta h \cos v + \Delta H \cos V + \text{small corrections of the second and higher orders,}$$

and where

d the true distance,
d' the apparent distance,
v the angle at the Moon's apparent place,
V the angle at the other object's apparent place,
Δh the correction for the Moon's altitude,
ΔH the correction for other object's altitude.

By transforming (1) we have

$$d = d' - \Delta h\,(1 - 2\sin^2 \tfrac{1}{2}v) + \Delta H\,(1 - 2\sin^2 \tfrac{1}{2}V) + \ldots\ ,$$

or

$$(2)\qquad d = d' - (\Delta h - \Delta H) + 2\,(\Delta h \sin^2 \tfrac{1}{2}v - \Delta H \sin^2 \tfrac{1}{2}V) + \ldots\ .$$

Now

$$\sin^2 \tfrac{1}{2}v = \frac{\cos \tfrac{1}{2}s \sin(\tfrac{1}{2}s - H')}{\sin d' \cos h'}$$

$$= \frac{\cos \tfrac{1}{2}s \sin(\tfrac{1}{2}s - H') \cos(\tfrac{1}{2}s - d')}{\sin d' \cos h' \cos(\tfrac{1}{2}s - d')},$$

where

h' the Moon's apparent altitude,
H' the other object's apparent altitude, and
$s = h' + H' + d'$.

But

$$\frac{\sin(\tfrac{1}{2}s - H')}{\cos h' \cos(\tfrac{1}{2}s - d')} = \tan h' - \tan(\tfrac{1}{2}s - d'),$$

and

$$\frac{\sin d'}{\cos \tfrac{1}{2}s \cos(\tfrac{1}{2}s - d')} = \tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d').$$

1879MNRAS..39..425A

Consequently

$$(3)\qquad \sin^2 \tfrac{1}{2}v = \frac{\tan h' - \tan(\tfrac{1}{2}s - d')}{\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')}.$$

In the same manner

$$(4)\qquad \sin^2 \tfrac{1}{2}V = \frac{\tan H' - \tan(\tfrac{1}{2}s - d')}{\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')}.$$

Substituting these values of $\sin^2 \frac{1}{2}v$ and $\sin^2 \frac{1}{2}V$ in (2),

$$(5)\qquad d = d' - (\Delta h - \Delta H) + \frac{[\tan h' - \tan(\tfrac{1}{2}s - d')]\,\Delta h - [\tan H' - \tan(\tfrac{1}{2}s - d')]\,\Delta H}{\tfrac{1}{2}[\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')]} + \ldots$$

or, putting

$$\tan h' - \tan(\tfrac{1}{2}s - d') = A,$$
$$\tan H' - \tan(\tfrac{1}{2}s - d') = B,$$
$$\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d') = C,$$

this becomes

$$(6)\qquad d = d' - (\Delta h - \Delta H) + \frac{A\Delta h - B\Delta H}{\tfrac{1}{2}C} + \ldots$$

When (1) is transformed in the form

$$d = d' - \Delta h\,(2\cos^2 \tfrac{1}{2}v - 1) + \Delta H\,(2\cos^2 \tfrac{1}{2}V - 1) + \ldots,$$

or

$$(7)\qquad d = d' + (\Delta h - \Delta H) - 2\,(\Delta h \cos^2 \tfrac{1}{2}v - \Delta H \cos^2 \tfrac{1}{2}V) + \ldots,$$

an analogous deduction gives

$$(8)\qquad \cos^2 \tfrac{1}{2}v = \frac{\tan \tfrac{1}{2}s - \tan h'}{\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')},$$

$$(9)\qquad \cos^2 \tfrac{1}{2}V = \frac{\tan \tfrac{1}{2}s - \tan H'}{\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')}.$$

Consequently,

$$(10)\qquad d = d' + (\Delta h - \Delta H) - \frac{(\tan \tfrac{1}{2}s - \tan h')\,\Delta h - (\tan \tfrac{1}{2}s - \tan H')\,\Delta H}{\tfrac{1}{2}[\tan \tfrac{1}{2}s - \tan(\tfrac{1}{2}s - d')]} + \ldots;$$

or, putting

$$\tan \tfrac{1}{2}s - \tan h' = A',$$
$$\tan \tfrac{1}{2}s - \tan H' = B',$$

this is

$$(11)\qquad d = d' + (\Delta h - \Delta H) - \frac{A'\Delta h - B'\Delta H}{\tfrac{1}{2}C} + \ldots.$$

1879MNRAS..39..425A

Since $\Delta d = d - d'$ is always $< 1^\circ$, it is, in most cases, sufficiently correct to take the arcs to the nearest minute, and to perform the calculation with four-figure natural tangents.

When greater accuracy is required, especially when the altitudes are low and the distance small, the corrections of the second order (first deduced by Lexell*) may be taken into account, viz.

$$(12)\quad \left\{\begin{array}{l} \tfrac{1}{2}(\Delta h^2 - \Delta d^2) \sin 1'' \cot d' \\ + \dfrac{\cos(h' + H')\cos(h' - H')}{\cos h' \cos H'} \Delta h \Delta H \sin 1'' \operatorname{cosec} d' \\ + \tan h' \tan H' \Delta h \Delta H \sin 1'' \cot d' \\ + \tfrac{1}{2}\Delta H^2 \sin 1'' \cot d'. \end{array}\right.$$

The first of these corrections, commonly called "the third correction," is contained in Chauvenet's *Spherical and Practical Astronomy*, ii. pp. 620, 621, as well as in almost all collections of nautical tables, and is thence easily applied. The other three corrections are, in low latitudes, generally very small, but in high latitudes cases are common in which the sum of these corrections is $> 10''$.

According to Legendre† the sum of all four corrections of the second order can be taken into account when Δd is recalculated with $d' + \frac{1}{2}\Delta d$, $h' + \frac{1}{2}\Delta h$, and $H' - \frac{1}{2}\Delta H$, viz. the given variables augmented with their half variations.

Example:

$$d' = 51^\circ\ 24'\ 45'',\quad h' = 58^\circ\ 56',\quad H' = 62^\circ\ 45',\quad \Delta h = 30'\ 4'' = 1804'',\quad \Delta H = 26''.$$

By (6).

° ′				
51 25				
58 56	1·6599			
62 45		1·9416		
173 6				
86 33				16·5874
35 8	0·7037	0·7037		0·7037
	0·9562	1·2379		15·8837
	1804	26		
				° ′ ″
				51 24 45
	$\dfrac{1725-32}{7\cdot942} = 213'' =$		+	3 33
			−	29 38
			$d =$	50 58 40

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* *Act. Acad. Petr. p. A.* 1777. Petropoli, 1780, p. 348.

† *Mém. de l'Institut*, t. iv., 1805; and Delambre's *Astronomie*, t. iii., 1814, p. 625.

1879MNRAS..39..428.

By (11).

° ′			
51 25			
58 56	1·6599		
62 45		1·9416	
173 6			
86 33	16·5874	16·5874	16·5874
35 8			0·7037
	14·9275	14·6485	15·8837
	1804	26	

$$\begin{array}{rrrr} & \circ & \prime & \prime \\ & 51 & 24 & 45 \\ -\frac{26929-381}{7\cdot942} = -3343'' = - & & 55 & 43 \\ + & & 29 & 38 \\ d = & 50 & 58 & 40 \end{array}$$

The "third correction" is here only 1″·6, and the other corrections are evanescent.

The following formulæ are of the same class as (3), (4), (8), and (9):

$$\cos A = \frac{\cot(\tfrac{1}{2}s-b)+\cot\tfrac{1}{2}s-2\cot c}{\cot(\tfrac{1}{2}s-b)-\cot\tfrac{1}{2}s} = \frac{\cot(\tfrac{1}{2}s-c)+\cot\tfrac{1}{2}s-2\cot b}{\cot(\tfrac{1}{2}s-c)-\cot\tfrac{1}{2}s},$$

$$\text{vers sin } A = 2.\frac{\cot b-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-c)-\cot\tfrac{1}{2}s} = 2.\frac{\cot c-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-b)-\cot\tfrac{1}{2}s},$$

$$\sin\tfrac{1}{2}A = \sqrt{\frac{\cot b-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-c)-\cot\tfrac{1}{2}s}} = \sqrt{\frac{\cot c-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-b)-\cot\tfrac{1}{2}s}},$$

$$\cos\tfrac{1}{2}A = \sqrt{\frac{\cot(\tfrac{1}{2}s-b)-\cot c}{\cot(\tfrac{1}{2}s-b)-\cot\tfrac{1}{2}s}} = \sqrt{\frac{\cot(\tfrac{1}{2}s-c)-\cot b}{\cot(\tfrac{1}{2}s-c)-\cot\tfrac{1}{2}s}},$$

$$\tan\tfrac{1}{2}A = \sqrt{\frac{\cot b-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-c)-\cot b}} = \sqrt{\frac{\cot c-\cot\tfrac{1}{2}s}{\cot(\tfrac{1}{2}s-b)-\cot c}},$$

where A is one of the angles of a spherical triangle, b, c the adjacent sides, and s the sum of the sides.

On the Spectrum of Brorsen's Comet, observed at the Royal Observatory, Greenwich.

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Brorsen's Comet was examined with the half-prism spectroscope mounted on the Great Equatoreal on April 17, and subsequent evenings whenever the state of the sky allowed. The dispersion used was that of one compound "half-prism," equivalent to four flint prisms of 60° (20° from A to H) with a power of 12 on the viewing telescope.